Downeast Rapid Transit Authority

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Towards an Aesthetic Kinematics of Subway Motion

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Abstract

In an effort to develop a comfortable and beautiful rider experience on the DRT, we propose five aesthetic and physical constraints to the trains' motion. We analyze the kinematic implications of these constraints and find that a uniform inter-station travel time of 120 secs ("two minutes per trip") can be achieved by requiring that adjacent stations be separated by no more than 16km. This elegant solution will require some modification of the existing DRT system route map, most notably the addition of several stations to interrupt the long inter-station distances on the Barrens, Lake, and Peninsula Lines.

Constructing an underground transit system from scratch in a rural area with an ample budget (see DRT Technical Note #2015-0411) offers myriad opportunities for innovation in transportation system design. Chief among these is the opportunity to give thoughtful attention from the outset to designing subway timetables that maximize rider comfort, convenience, and aesthetics. This Technical Note outlines a proposed scheme that simultaneously provides (a) remarkably smooth train motion (b) the scheduling convenience of uniform inter-station travel-times (despite wide variations in inter-station distances) and (c) a pleasing aesthetic symmetry of motion between all legs of a long subway trip. A trip between any two adjacent stations will take just as long and feel as smooth as one between any other two adjacent stations. In short, a ride on the D will be a thing of beauty.

To develop this scheme of maximally convenient, comfortable, and beautiful train travel, we impose the following kinematic constraints:

1. **Trains shall operate at subsonic velocities.** As vehicle velocities approach the trans-sonic region, aerodynamic heating and drag can lead to modes of mechanical instability not present at slower speeds. To avoid the complex engineering solutions needed to mitigate these effects, we propose limiting train speeds to below Mach 0.8 (about 250m/s or 600mph).
2. **Maximum horizontal accelerations shall be limited to 1.0g**. Riding the "D" should be a relaxing and enjoyable social event, not an extreme sport. Although many other modern transportation systems routinely generate brief periods of large accelerations (e.g., an automobile accelerating from 0-60 mph in 10 seconds (2.7g), Space Shuttle launch and reentry (3g), high-g roller coasters (>3.5g), F16 aircraft pulling out of a dive (8g))[1], we propose limiting accelerations to well below these extremes. We therefore recommend that total horizontal accelerations (the magnitude of the vector sum of lateral and longitudinal components) not exceed 1.0g.
3. **Jerk shall not exceed 0.35 m/s^3.** Mechanical jerk (the time-derivative of acceleration) plays the most crucial role in riders' perception of the "smoothness" of a ride.[2] Following the general principles of railway design, we recommend track alignments, carriage tilts, and dynamic acceleration profiles that constrain the jerk to under 0.35 m/s^3. Furthermore, we recommend the design, development, and installation of pneumo-magnetic jerk-dampers to further constrain the jerk to under 0.15 m/s^3, resulting in one of the smoothest known train rides.
4. **Velocity profiles shall be identical between all stations**. There is a pleasing aesthetic symmetry to requiring that the shape of the velocity profile (train velocity *vs* time) for all inter-station trips be congruent. Under this constraint, the overall magnitude (height) and duration (width) of the velocity curves would, of course, vary according to the distance between stations.
5. **The travel time between any two adjacent stations shall be constant.** With suitable selection of velocity profiles, it is possible to design a timetable that is easy for customers to understand and remember.

We begin the analysis by modeling a train traveling on a straight-line path, following a cosine velocity profile V(t):

$$V = \frac{1}{2} V_{max}\left(1 - \cos\frac{2\pi t}{T}\right) \qquad \text{(i)}$$

where t is the time, T is the total travel time between adjacent stations, and V_{max} is the maximum velocity attained during the trip. By design, this profile offers a smoothly varying velocity that climbs sinusoidally up from zero, attains V_{max} at the midpoint of the trip, then slows back down to zero upon arrival at the destination. By requiring the same velocity profile (i) in each inter-station trip, kinematic constraint **4**, above, is met. Similarly, requiring that T be constant for each trip satisfies kinematic constraint **5**.

From (i) the travel distance (or displacement, *S*), the acceleration (*a*), and the jerk (*j*) are easily derived:

$$S = \frac{1}{2} V_{max} \left(t - \frac{T}{2\pi} \cos\left(\frac{2\pi t}{T} \right) \right) \qquad \text{(ii)}$$

$$a \equiv \dot{V} = \frac{\pi}{T} V_{max} \sin\left(\frac{2\pi t}{T} \right) \qquad \text{(iii)}$$

$$j \equiv \ddot{V} = \frac{2\pi^2}{T^2} V_{max} \cos\left(\frac{2\pi t}{T} \right) \qquad \text{(iv)}$$

The relationship between these quantities is best visualized graphically. Figure 1 shows equations (i)-(iv) plotted for a trip of 120 seconds duration—that is, for a system in which a trip between any two adjacent stations takes two minutes, no matter how far apart those stations are. Distance, speed, acceleration, and jerk are shown in the figure for three representative inter-station distances: 1km, 10km, and 20km.

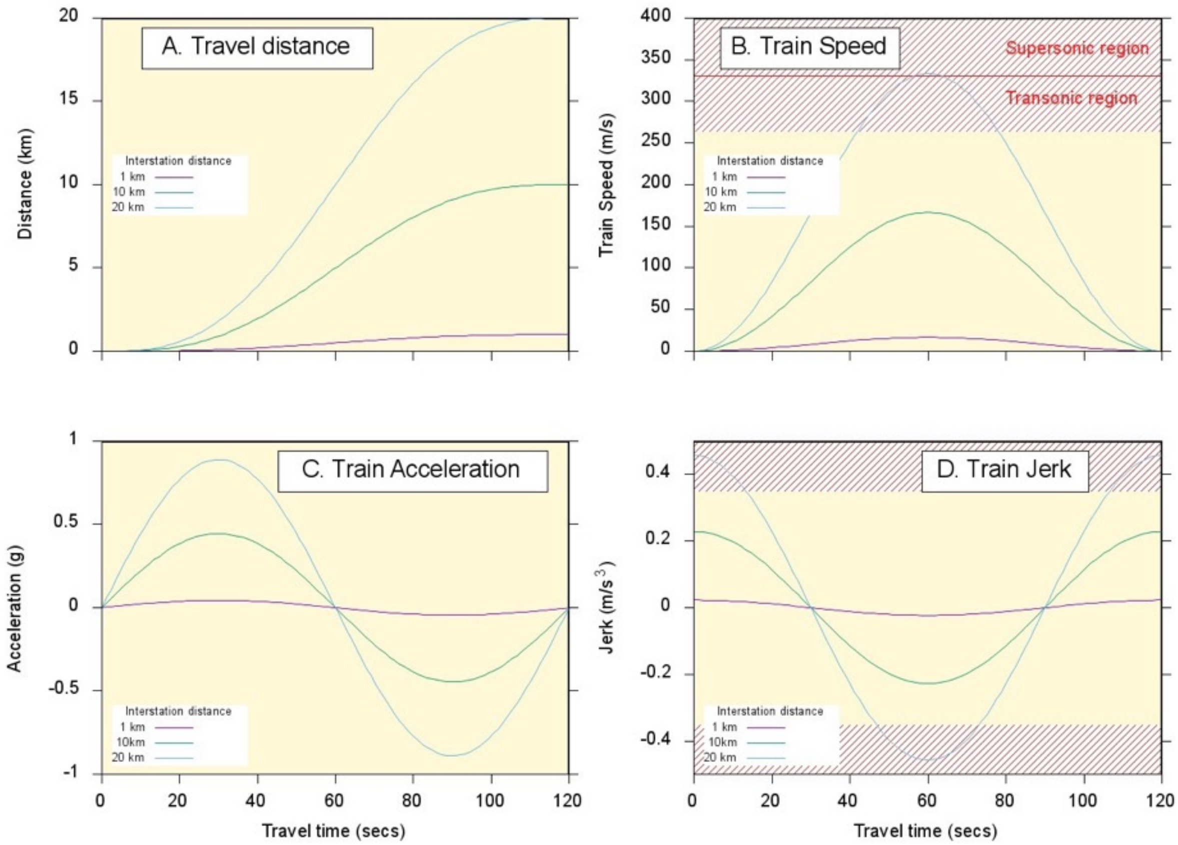

Figure 1. Kinematics of straight-line train motion following the cosinusoidal velocity profile of equation (i). Each panel in the figure spans a trip of duration 120 seconds between adjacent stations, for inter-station distances of 1km, 10km, and 20km. Panel A: cumulative travel distance (km). Panel B: forward train velocity ("speed"); the solid red line indicates the speed of sound in air; hatched area is forbidden by constraint **1**. Panel C: longitudinal train acceleration. Panel D: jerk (time-derivative of acceleration); hatched areas are forbidden by constraint **3**. In each panel, note the similar shape of the three curves, in keeping with aesthetic/kinematic constraint **4**.

Our task now is to discover how the kinematic constraints affect station spacings and inter-station travel times across the proposed DRT system. To this end, we reparameterize the constraints in terms of inter-station distance, S, and travel time, *T.* The peak values of the quantities in (i)-(iv) may be conveniently expressed in these terms*:*

$$V_{max} = (2/T)\,S$$
$$a_{max} = (2\pi/T^2)\,S \qquad \text{(v)}$$
$$j_{max} = (4\pi^2/T^3)\,S$$

The kinematic constraints **1**, **2**, and **3** may then be written as:

1. $S < ½\, 0.8cT$ (where c is the speed of sound in air; typ. 330 m/s)
2. $(2\pi/T^2)\,S < 1g$ (vi)
3. $(4\pi^2/T^3)S < 0.35$ m/s^3

Figure 2 shows the time-varying kinematic parameters plotted in *S-T* space. The kinematically "acceptable" region lies in the green hatched area below the three curves.

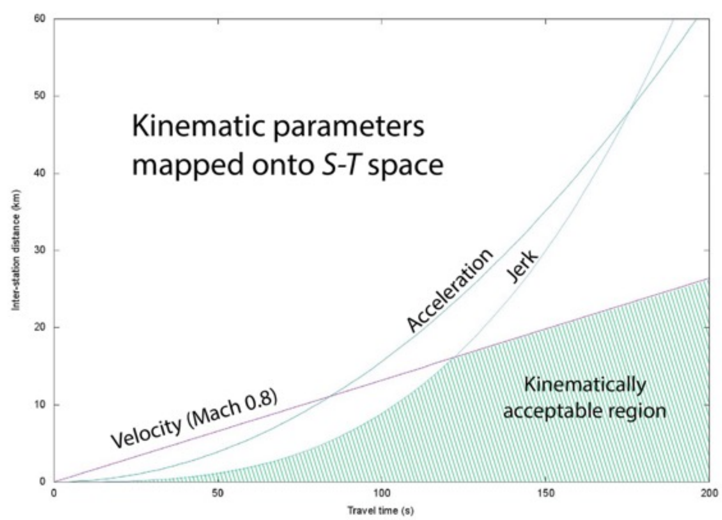

Figure 2. Velocity, acceleration, and jerk parameterized in terms of inter-station separation *S* and travel time *T*. In the region below each curve the kinematic constraints (vi) are met. The hatched area is the region where all constraints are simultaneously met.

There is a critical inter-station separation, S_c, below which acceptable motion is jerk-limited and above which it is velocity-limited. The transition between these two regimes occurs when the travel time satisfies the condition

$$T_c^2 = 2\pi^2 V_c / j_c \qquad \text{(vii)}$$

where T_c is the "critical" travel time (the travel time at which motion becomes velocity-limited), V_c is the "critical" velocity (the maximum permissible speed, Mach 0.8) and j_c is the "critical" jerk (maximum permissible jerk, 0.35 m/s^3). This yields T_c =122s and S_c =16km. In other words, as long as adjacent stations are separated by no more than 16km, we can guarantee that a trip between any two adjacent stations will take exactly two minutes.

Two minutes per trip is an ideal duration for a one-stop subway ride. It is just long enough for passengers to settle in to enjoy a ride of world-class smoothness and symmetry, while not so long that passengers will feel burdened by a long and interminably slow ride, with the inevitable thoughts of, "I should have taken the car."

Unfortunately, several stations on the current DRT system route map are spaced considerably farther apart than the critical separation of 16km. The Barrens, Lake, and Peninsula Lines are particularly prone to this problem. (Calais to Eastport is the longest leg, at 34.4km.) One solution, of course, would be to increase the inter-station travel time, T, across the entire system, thereby ensuring subsonic travel on even the longest legs. At S_c=35km, this would require a minimum travel time of T_c =265s. With this velocity profile, a one-way end-to-end trip on DRT's longest line (the Peninsula Line, with 16 stops from Sorrento to Calais) would take 71 minutes, plus total station dwell time (at least 15 minutes). This is comparable to the time required to make the trip by automobile, a situation incompatible with the "high-speed" requirement of DRT's mission. We therefore recommend preserving the "two minute per trip" constraint and finding an alternative solution to the velocity-limit problem.

We now consider several possible solutions to the velocity-limit problem:

1. Abandon constraint **1** by allowing supersonic travel on the longer legs. This would require designing trains, trackbeds, and associated infrastructure that support supersonic motion. Although technically possible, we reject this approach in favor of honoring the aesthetics of our original kinematic constraints.
2. Abandon kinematic constraint **5** (uniform inter-station travel time), and let constraint **1** (the subsonic condition) impose a system-wide absolute speed limit. Again, we reject this on purely aesthetic grounds.
3. Depressurize the longer segments (>16km) of the tunnel in order to reduce aerodynamic heating and drag, and to reduce shockwave formation, thereby increasing V_{max}.. The engineering challenges of such a scheme are daunting, requiring new technologies for airlocks, gaskets, and train couplings to allow trains to transition smoothly between tunnel

segments of widely different pressurizations. We reject this solution because of its reliance on unknown and untested technologies.

4. Insert intermediate train stops so as to bring the maximum inter-station distance to within S_c=16km. We recommend this solution as the most reliable and cost-effective approach.

Limitations

It must be stressed that the present analysis is limited to straight-line motion. Although lateral accelerations can be expected when pulling out of or approaching some stations, these turns are generally made at the "tails" of the velocity curve, and will therefore contribute minimally to the total acceleration. With careful design of track turning radii and with implementation of active-tilt mechanisms on train carriages, the kinematic effects of turns can be made negligible.

Conclusions

The five kinematic and aesthetic constraints on train motion that we propose here will result in a rider experience that is at once comfortable, consistent, beautiful, and unique in the history of rapid transit. To achieve these goals, a slight modification of the existing DRT route map will be required—namely, the introduction of additional stations so as to keep inter-station separations below 16km.

We will refer our findings to both the Cartographic Department and the Routing Department. We recommend that their determinations of new station locations be referred in turn to the Finance Department for final approval. The introduction of new stations will, of course, require the revision of existing route maps, signage, timetables, etc., all of which fall within the purview of the Chronometry and Design Departments.

Dynamical considerations of train motion will be addressed in a forthcoming Technical Note.

Notes

[1] "Orders of magnitude (acceleration)", Wikipedia, http://en.wikipedia.org/wiki/Orders_of_magnitude_(acceleration), retrieved 20150605.

[2] See "Jerk (physics)", Wikipedia, http://en.wikipedia.org/wiki/Jerk_%28physics%29 , retrieved 20150606.